Groundwater

Application of an Analytical Solution as a Screening Tool for Sea Water Intrusion

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Abstract

Sea water intrusion into aquifers is problematic in many coastal areas. The physics and chemistry of this issue are complex, and sea water intrusion remains challenging to quantify. Simple assessment tools like analytical models offer advantages of rapid application, but their applicability to field situations is unclear. This study examines the reliability of a popular sharp-interface analytical approach for estimating the extent of sea water in a homogeneous coastal aquifer subjected to pumping and regional flow effects and under steady-state conditions. The analytical model is tested against observations from Canada, the United States, and Australia to assess its utility as an initial approximation of sea water extent for the purposes of rapid groundwater management decision making. The occurrence of sea water intrusion resulting in increased salinity at pumping wells was correctly predicted in approximately 60% of cases. Application of a correction to account for dispersion did not markedly improve the results. Failure of the analytical model to provide correct predictions can be attributed to mismatches between its simplifying assumptions and more complex field settings. The best results occurred where the toe of the salt water wedge is expected to be the closest to the coast under predevelopment conditions. Predictions were the poorest for aquifers where the salt water wedge was expected to extend further inland under predevelopment conditions and was therefore more dispersive prior to pumping. Sharp-interface solutions remain useful tools to screen for the vulnerability of coastal aquifers to sea water intrusion, although the significant sources of uncertainty identified in this study require careful consideration to avoid misinterpreting sharp-interface results.

Introduction

Sea water intrusion is of concern in many coastal aquifers around the globe (Ferguson and Gleeson 2012; Werner et al. 2013), yet assessing the likelihood of groundwater withdrawals causing sea water intrusion to reach pumping wells is not straightforward. For example, the simulation of variable-density groundwater flow and solute transport in transient and heterogeneous coastal groundwater systems is computationally expensive, requires extensive field data for calibration, and is not always successful (Sanford and Pope 2010). Data scarcity and limits to computer-modeling capabilities have prompted a search for simpler management tools. The simplicity of indexing models such as GALDIT (Chachadi and Lobo-Ferreira 2001) is an attractive alternative to more complex approaches, but these indexing approaches are subjective and lack a theoretical foundation. The physics contained in analytical solutions, and the simplicity of their application, may provide a good balance between accounting for the relevant processes, data requirements, and computational efficiency (Werner et al. 2012; Morgan and Werner 2014).

There are a number of analytical models for predicting the position of the fresh water–salt water interface in coastal aquifers. These can be generally classified according to whether they consider regional groundwater flow (Glover 1959; Kooi and Groen 2001) and/or the effect of a pumping well (Reilly and Goodman 1987; Motz 1992). The solution developed by Strack (1976) is of particular interest in the context of groundwater management because it accounts for both regional groundwater flow and pumping (Figure 1). This solution is less flexible than numerical approaches because it applies only to homogeneous, isotropic, steady-state settings with simple aquifer geometries and assumes a sharp interface between fresh water and sea water. Despite these simplifying assumptions, other studies have suggested that Strack's (1976) solution offers useful insights into coastal aquifer conditions (Mantoglou 2003; Park et al. 2009). However,

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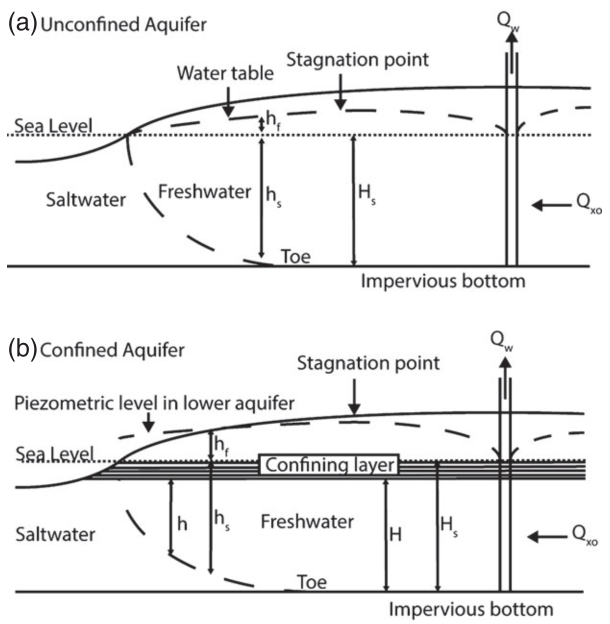

Figure 1. The geometry of the Strack (1976) solution for the steady-state position of the salt water interface under (a) unconfined conditions and (b) confined conditions. Pumping of the well causes a local depression in hydraulic head, and a stagnation point forms at the potentiometric high between the well and the coast.

Strack's analytical model has not been rigorously tested to explore the method's robustness under a variety of field-scale conditions, which have more complex boundary conditions and material property distributions than the model.

The effect of neglecting the dispersive nature of the fresh water–salt water interface, along with other simplifying assumptions of the model, on the efficacy of Strack's (1976) solution applied to field conditions is not clear. In this study, the potential for using this solution as a tool for obtaining initial indications of allowable pumping rates is explored in the context of coastal aquifer management.

Testing of sharp-interface solutions is somewhat more challenging than testing dispersive modeling approaches because the latter predict the spatial distribution of a continuum of salt concentrations that can be readily compared to observed salinities. Agreement between the model and observations can then be assessed with statistical measures such as the coefficient of determination or root-mean-square error (Anderson and Woessner 1992; Hill 1998). The subset of these data that is available to test Strack's (1976) solution is far smaller as only the sharp interface, often approximated by the isochlor representing 50% sea water, is produced by the model. In many cases, it will only be possible to compare model output to a few values that are usually based on interpolation between salinity measurement points. An alternative is to examine the performance of Strack's (1976) solution in a number of settings using categorical statistics (Fielding and Bell 1997; Landis and Koch 1977). These statistics are more suited to cases where a solution indicates the presence or absence of some entity or event, which in this case is the contamination of a pumping well due to sea water intrusion. Here, the effectiveness of the original Strack (1976) solution and a modified version to account for dispersion (Pool and Carrera 2011) are tested against production-well salinities in Australia, Canada, and the United States. The results are examined to determine the possible reasons for errors, biases, and uncertainty.

Theory

The position of the fresh water–salt water interface in coastal aquifers is most commonly controlled by the difference in density between fresh water and sea water and the opposing flow of fresh groundwater towards the sea. Strack (1976) used the method of potentials to develop a steady-state solution to this problem that considers both regional groundwater flow as discharge per unit length of coastline Q_{xo} (m^2/s) and groundwater extraction Q_w (m^3/s) from a fully penetrating well. The solution assumes negligible vertical flow and a sharp interface between fresh water and sea water that intersects the coast. The aquifer considered in this analytical model has a base at a constant depth H (m) below sea level and extends an infinite distance both inland and parallel to the coast.

The following equations were developed to establish whether a well at a distance of x_w (m) from the coast would experience sea water intrusion (Strack 1976):

$$\lambda = 2\left[1 - \frac{\mu}{\pi}\right]^{1/2} + \frac{\mu}{\pi}\ \ln\left(\frac{1-(1-\mu/\pi)^{1/2}}{1+(1-\mu/\pi)^{1/2}}\right) \quad (1)$$

where

$$\mu = \frac{Q_w}{Q_{xo}x_w} \quad (2)$$

$$\lambda = \frac{KH^2}{Q_{xo}x_w}\varepsilon \quad (3)$$

and K is hydraulic conductivity (m/s); ρ_s is the density of sea water (kg/m^3); ρ_f is the density of fresh water (kg/m^3); $\varepsilon = (\rho_s/\rho_f^2)(\rho_s - \rho_f)$ for confined aquifers; and $\varepsilon = (\rho_s - \rho_f)/\rho_f$ for unconfined aquifers. μ and λ are plotted against each other to form what Strack (1976) describes as the "stability line" (Figure 2). Values that plot above or to the right of the stability line indicate that the well should experience sea water intrusion once steady-state conditions have been achieved.

Examination of (1) reveals that values of $\mu > \pi$ and $\lambda > 2$ should always result in sea water intrusion to a pumping well. In cases where $\mu > \pi$, there are no values of λ that will prevent sea water intrusion because the capture zone of the well extends offshore. Sustainable production of groundwater will only be possible if the pumping rate is reduced to produce a value of $\mu < \pi$. Instances where $\lambda > 2$ imply that the well has been completed in a saline portion of the aquifer prior to any groundwater extraction (Strack 1976). The well must

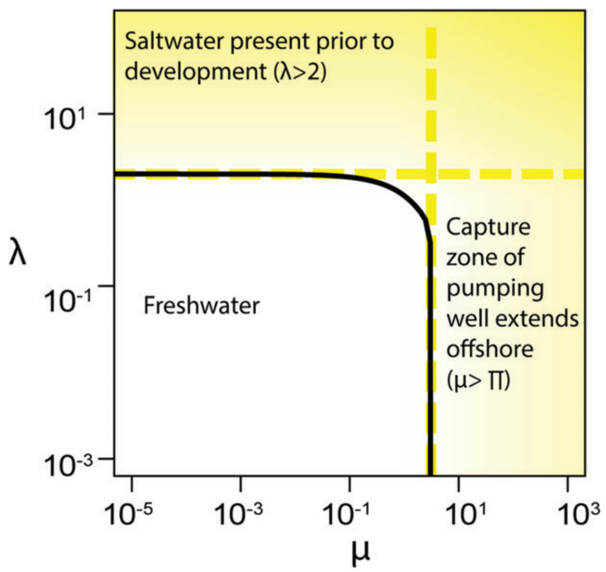

Figure 2. Plot of μ and λ showing positions of fresh water and salt water due to predevelopment conditions, pumping, and a combination of those two factors.

be moved inland to allow for any pumping to occur, notwithstanding the ability of partially penetrating wells to draw on fresh water occurring above salt water. To prevent sea water intrusion at a pumping well, values of both μ and λ need to be slightly less than these maximum values in the curved area of the stability line (Figure 2), occurring near $\mu = \pi$ and $\lambda = 2$.

Pool and Carrera (2011) created a series of variable-density numerical models to assess the impacts of dispersion and related hydrodynamic processes on the application of Equation 3. The Pool and Carrera (2011) correction, accounting for dispersion effects, results in a seaward shift of the sea water wedge toe. This leads to higher pumping rates before wells experience salinization. For sea water intrusion resulting in a salinity of 0.1% of sea water in a pumping well, Pool and Carrera (2011) propose the use of ε^* rather than ε in Equation 3, where:

$$\varepsilon^* = \varepsilon \left[1 - \left(\alpha_T/H\right)^{1/6}\right] \tag{4}$$

where α_T is transverse dispersivity (m). In this study, we use Equation 3 in both its original and modified forms to assess the effectiveness of this correction. Other studies have considered alternative values for the exponent (1/6) in Equation 4, depending on the salinity contour used (Lu and Werner 2013) and the geometry of the seepage face associated with fresh water discharge at the shoreline (Koussis et al. 2015).

Testing Against Field Data

Data relating to aquifers from eight coastal regions (Figure 3), representing a range of conditions, were compiled to test Equations 1 through 3 (Table 1). These regions include the eastern shore of Virginia (Sanford et al. 2009), the Floridan aquifer in Florida (Andersen et al. 1988), Georgia (Warner et al. 1999) and South Carolina (Payne 2010), the Seaside Groundwater Basin of California (Yates et al. 2005), several locations in Nova Scotia, Canada (Beebe 2011), and five aquifers in various locations in Australia (Brown et al. 2006; Stewart 2007; Ivkovic et al. 2013). In total, 78 production wells were tested using Strack's analytical model.

H, x_w, K, and Q_w were estimated from a combination of data derived from physical investigations, published values, and existing hydrogeological models. In wells penetrating multiple aquifers, the average hydraulic conductivity was taken from the geometric mean of the aquifer units. Aquifer thickness was estimated as the combined thickness of the units. This type of layered system was only represented by a few locations from the eastern shore of Virginia. Q_{xo} was calculated from estimated hydraulic gradients, H and K values. Wells were chosen such that they were not expected to experience significant interference effects from nearby wells.

The estimated parameters (Table 1) were used to calculate μ and λ for 78 wells using Equation 3 in its original form (Figure 4a). Pool and Carrera's (2011) correction (Equation 4) was also used to calculate λ with α_T of 0.1 m, 1.0 m, and 10.0 m The values of μ and λ for pumping wells, shown in Figure 4, were evaluated against whether sea water intrusion had occurred at those wells, as defined by a salinity of 350 mg/L (1% sea water salinity).

Using the original form of Equation 3, 29 wells plotted in the zone of Figure 4a where sea water intrusion is predicted, including 22 where $\lambda > 2$ and 15 where $\mu > \pi$. Four wells had both $\lambda > 2$ and $\mu > \pi$. Of the 29 wells where sea water intrusion was predicted, 18 had elevated total dissolved solids or chloride levels. Correct predictions were made for 15 of the 22 cases where $\lambda > 2$, namely in Busselton, Derby, McLaren Vale, Port MacDonnell, and Virginia. Correct predictions were made for 6 of the 15 cases where $\mu > \pi$, namely in Nova Scotia, Florida, and Virginia. In the four cases where both $\lambda > 2$ and $\mu > \pi$, sea water intrusion was observed in three instances. It should be noted that the maximum salinity of all wells examined in this study was less than 3000 mg/L. This value is considerably fresher than the TDS value of 17,500 mg/L used in the uncorrected version of Equation 3, although production wells in contact with the interface will produce water of mixed sea water–fresh water origins of lower salinity than the 50% isochlor due to the distribution of flow lines (Shi et al. 2011). The lack of high salinities in the cases examined in this study is a logical consequence of the continued use of these wells for fresh water supply. That is, wells that draw highly saline groundwater are likely to be quickly decommissioned, and therefore, it is essentially impossible to include active wells strongly impacted by sea water intrusion in the current analysis.

The uncorrected Strack (1976) equation produced correct predictions for 46 of 78 (59%) of the wells in this study (Table 2). Incorrect predictions included 17 false negatives and 15 false positives. Note that if 50% sea water salinity were used as the criterion for sea water

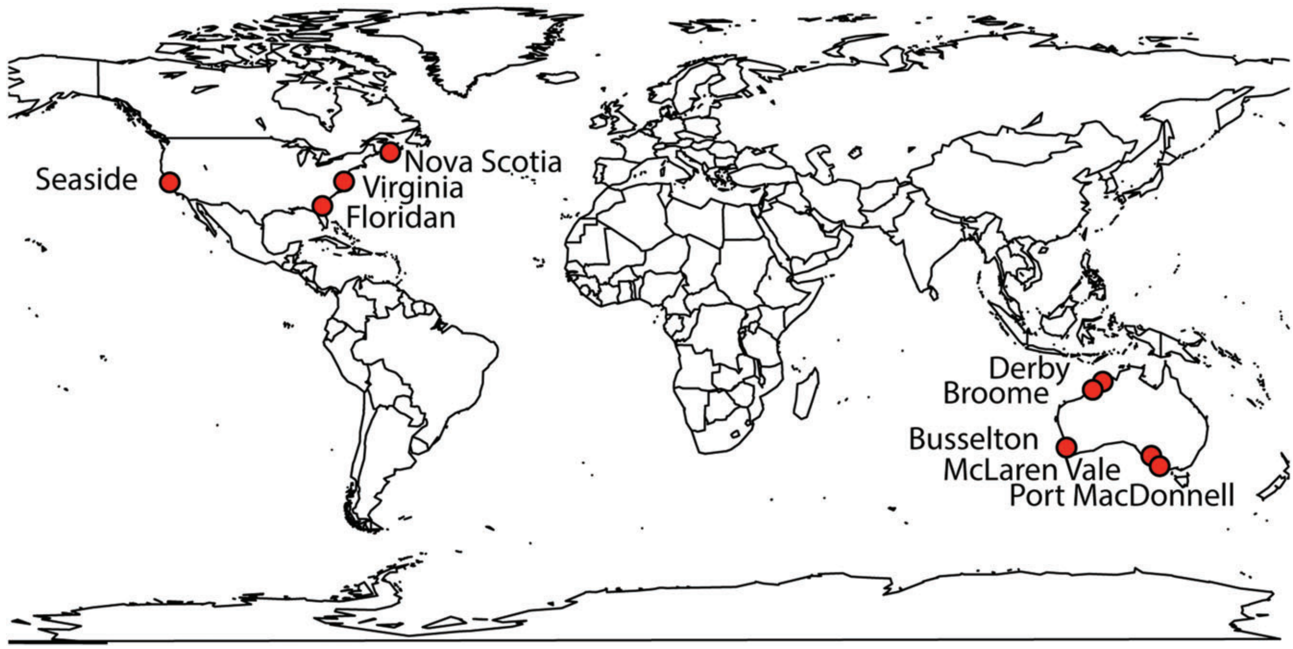

Figure 3. Location of aquifers examined in this study.

Table 1
Description of Study Areas and Data Used for Input into Equations 1 Through 3

Region	Hydraulic Gradient	Hydraulic Conductivity (m/s)	Aquifer Thickness (m)	Confined or Unconfined	Reference
Eastern shore of Virginia, USA	10^{-4}	10^{-6} to 10^{-4}	1–70	Confined	Sanford and Pope 2010
Floridan Aquifer System, USA	4 x 10^{-4} to 5 x 10^{-3}	10^{-4} to 10^{-3}	40–200	Both	Andersen et al. 1988; Warner et al. 1999
Nova Scotia, Canada	10^{-6} to 10^{-4}	10^{-6} to 10^{-4}	Variable	Both	Beebe 2011
Port McDonnell, South Australia	1.5 x 10^{-3}	10^{-3}	~300	Unconfined	Brown et al. 2006
McLaren Vale, South Australia	4 x 10^{-4}	10^{-3}	~90	Confined	Stewart 2007
Seaside, California, USA	2 x 10^{-3}	4 x 10^{-3}	100	Confined	Yates et al. 2005
Derby, Western Australia	4 x 10^{-4}	2 x 10^{-3} to 4 x 10^{-5}	170–270	Unconfined	Ivkovic et al. 2013
Broome, Western Australia	3 x 10^{-3}	9 x 10^{-5} to 3 x 10^{-4}	175–270	Unconfined	Ivkovic et al. 2013
Busselton, Western Australia	4 x 10^{-4}	6 x 10^{-6} to 6 x 10^{-5}	0–5	Unconfined	Ivkovic et al. 2013

intrusion to the pumping well, correct predictions are arrived at in 56% of cases, with all errors being false positives. Use of Pool and Carrera's (2011) correction resulted in correct predictions in 52–54% of cases, depending on the α_T value used. Use of the correction decreased the rate of false positives and increased the rate of false negatives by moving the predevelopment interface position toward the coast (Figure 5). In some thicker aquifers, the redefined interface moved several hundred metres closer to the shoreline.

The statistical measure κ has been suggested as a measure of the effectiveness of presence/absence models (Fielding and Bell 1997) and is defined as:

$$\kappa = \left[A - (BC + DE)/N) \,/\, N - ((BC + DE)/N)\right] \tag{5}$$

where $A = a + d$; $B = a + c$; $C = a + b$; $D = b + d$; $E = c + d$; a is the number of true positives; b is the number of false positives; c is the number of false negatives; d is the number of true negatives; and N is the sample size (Table 2). For the uncorrected version of Strack's (1976) equation, $\kappa = 0.17$, which suggests "slight" agreement between the predictions and reality, which Landis and Koch (1977) define as $0 < \kappa < 0.20$. Estimates using Pool and Carrera's (2011) correction had κ values between 0.01 and 0.03, depending on the value of α_T used in the correction.

Strack's equation performed better for confined aquifers examined in this study. Using the uncorrected version of Strack's equation, the correct prediction was made in 64% of cases and $\kappa = 0.257$ using λ values alone. Correct predictions were made in 50% of the cases using

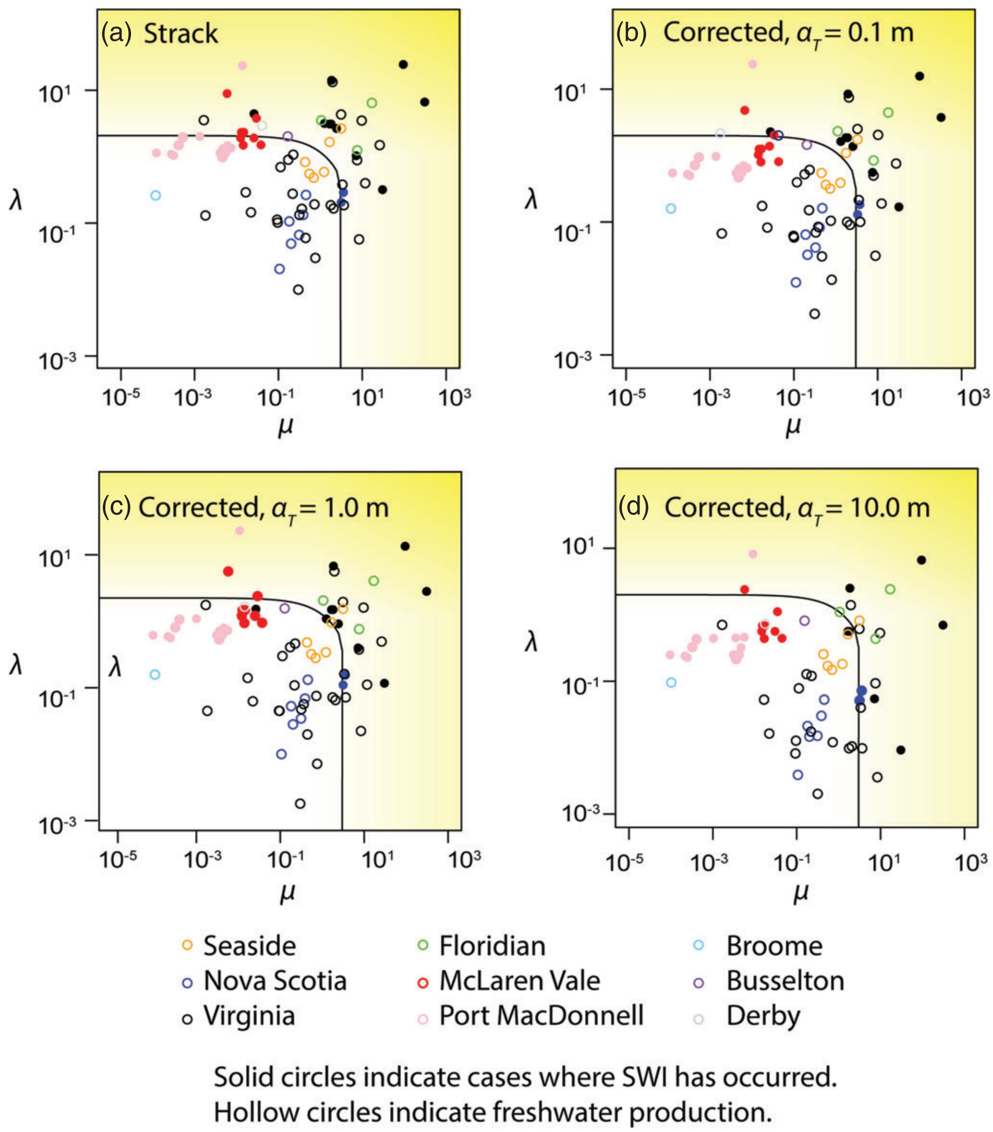

Figure 4. Plots μ and λ calculated for pumping wells examined in this study. The stability line (solid black) represents Equation 3. (a) Strack (1976), (b) corrected $\alpha_T = 0.1$ m, (c) corrected $\alpha_T = 1.0$ m, and (b) corrected $\alpha_T = 10.0$ m.

μ alone and $\kappa = -0.05$. Of all the cases examined, 15 were unconfined. Correct predictions were made in 33% of those cases, whereas correct predictions were made in 65% of confined aquifers.

The effectiveness of Strack's equation differed, depending on the position of the predevelopment interface (Figure 5). In the 47 cases where the predevelopment interface was less than 1000 m from the coast, correct predictions were made 76% of the time with $\kappa = 0.49$. In the 31 cases where the interface extended more than 1000 m inland, correct predictions were made in only 13% of cases and $\kappa = -0.26$.

The wells from McLaren Vale, Port MacDonnell, Broome, and Derby plot in the same general location in the upper left of the Strack plot (Figure 4), which indicates that these wells were installed relatively close to the predevelopment steady-state position of the interface. Successful predictions were made in only 6 of 24 cases from these areas. According to Strack's analytical model, the toe of the interface in those regions is expected to extend further inland than most of the other areas considered in this study (Figure 5). Salinities encountered at wells flagged as experiencing sea water intrusion were between 500 and 1000 mg/L in the Port MacDonnell area and up to 3500 mg/L in the McLaren Vale area (Government of South Australia 2014). Salinities in Broome and Derby were less than 350 mg/L. Busselton was the outlier among the Australian cases examined. The λ value indicates that this well was completed on the fresh water side of the predevelopment interface, but the μ value indicates that pumping at the reported rate will cause sea water intrusion once steady-state conditions are reached.

Wells from Virginia had large ranges in both μ and λ and produced 27 out of 37 correct predictions. The associated errors produced with the uncorrected version of Equation 3 were false positives (Figure 4). Extensive fresh groundwater that originated as Pleistocene recharge

Table 2
Results of Predictions Made Using Equations 1 Through 4

Observed		Predicted SWI (+)	Predicted No SWI (−)	Correct Predictions (%)	κ
Strack	SWI(+)	19	17	59.0	0.171
	No SWI (−)	15	27		
Corrected with $\alpha_T = 0.1$ m	SWI(+)	11	25	51.3	−0.004
	No SWI (−)	13	29		
Corrected with $\alpha_T = 1.0$ m	SWI(+)	9	27	52.6	0.012
	No SWI (−)	10	32		
Corrected with $\alpha_T = 10.0$ m	SWI(+)	9	27	53.8	0.037
	No SWI (−)	9	33		

SWI (+) indicates that the fresh water–sea water interface has reached the well, and SWI (−) indicates that the well remains on the fresh water side of the interface. Kappa is a measure of effectiveness for presence-absence models (see Equation 5).

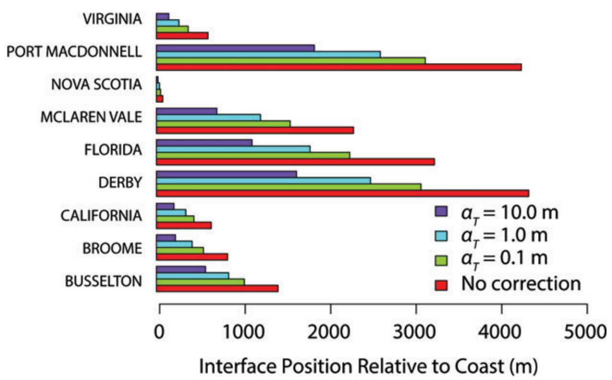

Figure 5. Average predevelopment steady-state position of the toe of the salt water–fresh water interface for the original Strack (1976) equation and the Pool and Carrera (2011) corrections for a range of transverse dispersivities for regions examined in this study.

has been noted in this region (Cohen et al. 2010). Given that the Strack (1976) solution presumes that the shoreline represents the limit of fresh water, it is likely that false positives in these regions are attributable to the offshore position of the fresh water–sea water interface, which allows wells to access fresh water stored in subsea aquifers prior to contamination by saltwater intrusion. The Floridan aquifer produces zero correct predictions for three wells in a similar area of Figure 4, all of which are false positives. The presence of older fresh water remaining offshore from past low sea levels (Morrissey et al. 2010) or outflow at significant distances offshore (Langevin 2003) could explain the presence of fresh water at uncorrected λ values between 1.2 and 6.2, as estimated for the wells in this aquifer.

Analysis using Equations 1 through 3 indicated that the wells examined in Nova Scotia should experience few instances of sea water intrusion. All wells had uncorrected $\lambda < 0.3$, and the presence or absence of sea water intrusion was predicted properly in all eight cases. This prediction is notable given the expected effects of till aquitards in this region, which tend to lead to fresh water in the offshore extensions of these aquifers (Beebe 2011). Unlike the aquifers on the Atlantic Coast of the United States, isotopic evidence suggests that groundwater in shallow confined aquifers in Atlantic Canada is modern rather than Pleistocene in age (Beebe 2011; Hansen 2012).

Wells from Seaside, California plot in a similar area of Figure 4 to those from Virginia, Florida, and Nova Scotia. Two false positives were found along with four correct predictions for wells that were very close to the stability line. A regional numerical model of this aquifer predicts that these wells will begin to produce saline water within the next century (Loáiciga et al. 2012), suggesting that the analytical approach here will produce the correct answer as the system approaches steady state.

Discussion

The overall performance of Strack's (1976) solution in the cases examined in this study is only "fair" according to guidelines provided by the categorical statistical analyses (Landis and Koch 1977). The uncorrected Strack (1976) equation produced an approximately equal number of false positives and false negatives (Table 2), and therefore, the unmodified Strack (1976) method at least lacked strong systematic bias. False positives are preferred over false negative errors where water quality is in question due to a commitment to the precautionary principle (Hrudey et al. 2006). The finding of both false negatives and false positives highlights that water resource managers need to rely on more sophisticated methods to assess whether sea water will invade a particular production well. Negative predictions may still require an evaluation of the threat of sea water intrusion if other evidence, such as field sampling, indicates that sea water intrusion is potentially impactful. The benefit of the method is that it provides a systematic basis for ranking sea water intrusion vulnerability, as outlined by Werner et al. (2012). However, on the face of the results, the overall distribution of false negatives and false positives arising from Strack's (1976) solution indicates that it is not conservative from a risk-aversion perspective.

The analysis here is complicated by biases arising from a lack of information on wells completed in the salt

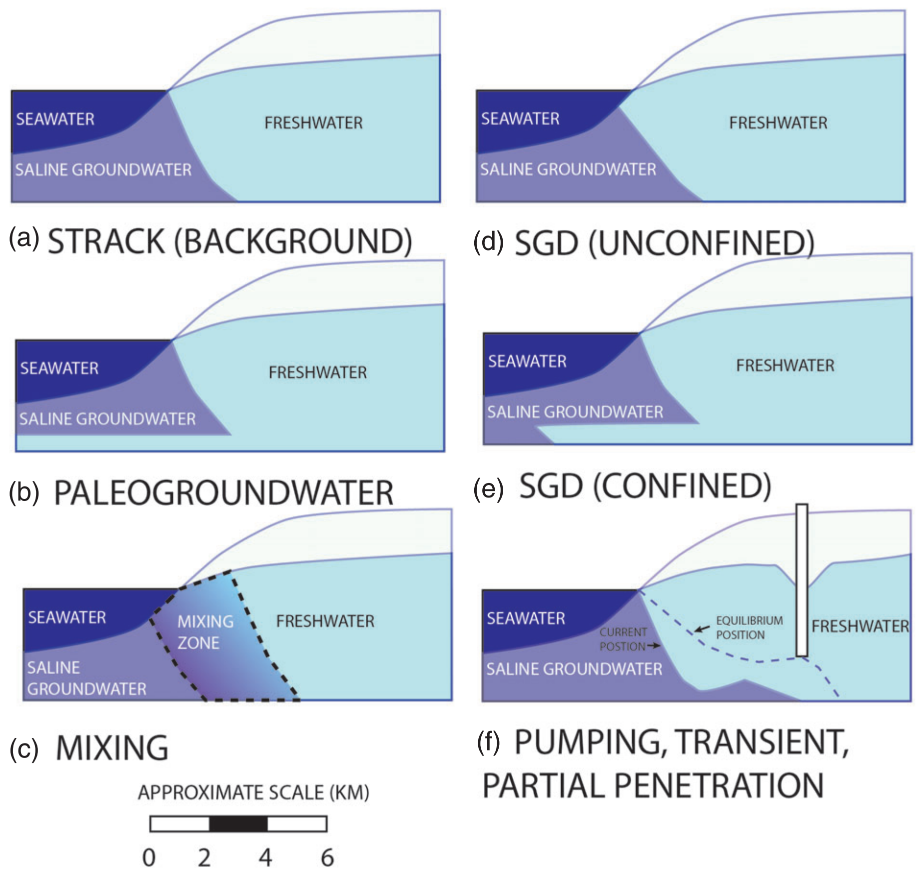

Figure 6. Factors leading to errors in (a) Strack's solution including: (b) presence of paleogroundwater; (c) dispersive mixing at the interface between freshwater and saline groundwater; (d) submarine groundwater discharge (SGD) for unconfined conditions; (e) SGD for confined conditions; and (f) effects of pumping.

water side of the interface, including production wells that are producing salt water. Wells encountering salt water are generally pumped in a manner that keeps salinity low and are decommissioned following sea water intrusion or left undeveloped if salt water is encountered during testing. Barring a large number of uncataloged saline wells in areas that this method would predict to be fresh, the method should be expected to produce more false positives than false negatives. However, this did not occur.

Inconsistencies between the simplifying assumptions made by the uncorrected version of Strack's (1976) equation and field conditions could create systematic errors resulting in bias. For example, the correction proposed by Pool and Carrera (2011) is intended to address the diffuse nature of the interface and the effect of seepage towards the coast (Figure 6c). Application of this correction did not improve the results and produced more false negatives. The majority (65%) of the cases where sea water intrusion is predicted using the uncorrected version of Strack's (1976) equation have values of $\lambda > 2$, indicating that it is not possible to obtain fresh water from a fully penetrating well at these locations, regardless of the pumping rate. Applying Pool and Carrera's (2011) correction shifted λ values downward relative to the uncorrected assessment, and 28–54% of predicted sea water intrusion cases had $\lambda > 2$, depending on the value of α_T used. However, the models produced by Pool and Carrera (2011) only considered one of the mechanisms responsible for dispersion in coastal aquifers. Mixing zones with widths ranging from a few metres to a few kilometres have been observed and are a function of geological heterogeneity, tidal fluctuations, and changes in sea level over time (Barlow 2003; Price et al. 2003; Paster et al. 2006) (Figure 6c). These more extensive mixing zones could result in salinity levels unsuitable for potable water supplies at wells that are considerably further inland than that predicted by the sharp-interface analytical model modified by the Pool and Carrera (2011) correction with typical values of α_T, on the fresh water side of the interface or leading to earlier than expected breakthrough of saline water at pumping wells.

The uncorrected version of Strack's (1976) equation tended to correctly predict SWI for cases where the wedge had smaller inland extent under predevelopment conditions. Where the predevelopment wedge extended less than 1 km inland, correct predictions were made in 76% of cases and $\kappa = 0.48$. The rates of successful prediction were the highest in Virginia, Nova Scotia, and California, where the wedge was expected to extend less than 1 km inland. Strack's (1976) equation performed poorly for wells in Port MacDonnell and McLaren Vale, Australia, where the wedge is expected to extend approximately

3 km inland under predevelopment conditions. We hypothesize that the failure of the sharp-interface model may be related to the degree of dispersiveness of the interface, whereby more extensive wedges are likely to have more dispersed interfaces. Studies examining the effect of dispersion on the distribution of salinity in salt water wedges have suggested that higher Peclet numbers (diffusive divided by advective fluxes) are associated with wider mixing zones (Abarca et al. 2007). Higher Peclet numbers are therefore associated with conditions that most deviate from sharp-interface predictions. Further, where the mixed convection ratio (defined as advective divided by density-driven forces) is small, sea water is more extensive in the aquifer (Abarca et al. 2007). Therefore, in situations where discharge to the sea is relatively small, the interface is both wider and found further inland. It follows that the performance of sharp-interface models will be the weakest for the most extensive sea water intrusion conditions, at least on the basis of dispersive effects, which are important in the current study and under real-world conditions more generally (Abarca et al. 2007; Pool and Carrera 2011). In other words, areas with more extensive sea water intrusion are expected to be more dispersive and therefore likely the hardest to quantify and predict.

The presence of fresh water offshore (Figure 6b) could be partially responsible for a number of the false positives found in this study. There are numerous examples where the position of the fresh water–salt water interface reflects lower sea levels experienced during the last glaciation, resulting in zones of fresh water tens of kilometres off the coast (Post et al. 2013) (Figure 6c). Significant areas of fresh water in submarine portions of aquifers can exist under equilibrium conditions as well (Stuyfzand 1995; Koussis et al. 2015) (Figure 6d and Figure 6e). Field studies have confirmed that measurable submarine groundwater discharge (SGD) is a common phenomenon over considerable areas at distances of several hundred metres from the coastline (Taniguchi et al. 2002). In areas where low permeability-confining layers exist, these zones could extend several kilometres from the coastline (Kooi and Groen 2001). However, the treatment of this problem by Strack (1976) assumes that all regional groundwater flow discharges at the coast under equilibrium conditions. The presence of offshore fresh water and transient conditions cause Equations 1 through 3 to predict false positives; that is, sea water intrusion is falsely predicted to occur. However, the analytical model may identify the long-term outcome of an aquifer once sub-sea fresh water is depleted.

Errors are probably introduced by the treatment of pumping in Strack's (1976) analytical model. The model, as applied here, does not consider cumulative drawdown effects, partially penetrating wells, or transient effects, such as intermittent pumping or seasonality (Figure 6f). Attempts were made to select pumping wells that would not be affected by nearby wells, but cumulative drawdown effects were not explicitly considered in this analysis. Neglecting well interference should lead to greater inland migration of saline water than expected (Park et al. 2009), resulting in false negatives with low λ values to the right of the vertical portion of stability line (Figure 4). No such wells were found in this study, and therefore, well interference effects are assumed to be relatively minor. Partially penetrating wells would be expected to cause false positives because the analytical model indicates whether the toe of the interface will reach the well's location. A partially penetrating well may continue to produce fresh water under these conditions if pumping rates are sufficiently low (Figure 6f). For most of the wells used in this study, the degree of aquifer penetration is unclear. However, the model does a marginal job of predicting salt water intrusion due to pumping. This is reflected by the low κ value of -0.05 for predictions based on μ alone. Some of the false positives could reflect non-steady-state conditions within a number of the systems (Figure 6f), including aquifers with offshore fresh water, as discussed above. In such cases, sea water intrusion may occur at some point in the future. Further analysis should be conducted to determine the time required to reach this equilibrium state. Such an analysis has been done for the Seaside aquifer (Loáiciga et al. 2012), and sea water intrusion is expected to occur within the next century for the two wells from this aquifer, which are unstable according to the Strack (1976) criteria. These wells are currently producing fresh water.

Non-marine sources of salinity were not considered in this study. Diagnosing such cases can be difficult without geochemical analyses. Examination of Br, Cl, and stable isotopes of oxygen and hydrogen along with other aspects of water chemistry is recommended to differentiate other sources of salinity (Stuyfzand and Stuurman 1994; Werner and Gallagher 2006). The absence of false positives for wells with low λ and μ values suggests that non-marine sources of salinity are not problematic in this study. However, some of the wells from Australia used in this study are situated in aquifers with high background (i.e. non-marine) salinities (Government of South Australia 2014). Further examination of water chemistry in other wells may reveal that other sources of salinity could contribute to actual incidences of sea water intrusion impacts at the wells used in this study.

Conclusions

The performance of Strack's (1976) analytical model along with the corrected version proposed by Pool and Carrera (2011) suggests that predicting sea water intrusion at an individual well by means of an analytical solution may not be feasible. A similar conclusion was reached by Sanford and Pope (2010) for numerical modeling techniques. Accurate parameter estimation is a problem in both studies, but additional problems related to an inability to handle complex initial and boundary conditions are encountered when applying analytical models. The influence of past low sea levels and fresh water offshore should allow for fresh water to be produced from many wells where sea water intrusion is predicted by Strack's (1976) solution. Furthermore, many wells could produce fresh water for long periods of time before the steady-state conditions described by Strack's solution are reached. Strack's (1976) solution also performed worse for aquifers with more extensive predevelopment salt water wedges, suggesting issues with dispersion. In cases where the predevelopment salt water wedge was expected to be less than 1 km long, Strack's (1976) solution produced the correct result in 39 of 52 (75%) cases.

While Strack's solution has only fair predictive capacity when compared against the entire dataset considered here, it could prove useful in assessing the sensitivity of a region to sea water intrusion. Given additional information about how actual aquifers differ from the idealized analytical model, it should provide some insight into the likelihood of sea water intrusion at a given well. The se of this solution as a theoretical basis to develop vulnerability indicators (Werner et al. 2012) is also supported by its ability to capture global trends and trends within individual aquifers. Further assessment of such indicators against field observations is required to understand the efficacy of this analytical solution.

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