Crop kites: Determining crop-water production functions using crop coefficients and sensitivity indices

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Abstract

The crop-water production function quantitatively evaluates the relationship between seasonal water use and crop yield and is used to evaluate optimal irrigation depth and assess the potential of deficit and supplemental irrigation. A simple and easily applicable methodology to develop crop- and region-specific crop-water production functions using crop coefficients and sensitivity-indices is presented. Previous efforts to describe the crop-water production function have not accounted for the effects of the temporal distribution of water use and trivialize the associated variability in yields by assuming an optimized or arbitrary temporal distribution. The temporal distribution of water use throughout the growing season can significantly influence crop yield, and the ability of farmers to manage both the timing and amount of irrigation water may result in higher yields. We propose crop kites, a tool that explicitly acknowledges crop yield as a function of the temporal distribution of water use to both evaluate the complete space of water use and crop yield relationships, and extract from this space specific crop-water production functions. An example for winter wheat is presented using previously validated crop-specific sensitivity indices. Crop-water production functions are extracted from the crop kite related to specific irrigation schedules and temporal distributions of water use. Crop-water production functions associated with maximizing agricultural production agree with previous efforts characterizing the shape as a diminishing curvilinear function. Crop kites provide the tools for water managers and policy makers to evaluate crop- and region-specific agricultural production as it relates to water management and the associated economics, and to determine appropriate policies for developing and supporting the infrastructure to increase water productivity.

1. Introduction

Agriculture demands more water than any other process in the world. Globally, ~20–30% of agricultural land is irrigated, with an associated water demand estimated at 80% of global water withdrawals and 90% of global water consumption (Wada et al., 2013). Increasing demands and the continued intensification of agriculture will likely continue to impact water resources and ecosystems (Bruinsma et al., 2015; de Fraiture et al., 2007; Feneres and Soriano, 2007; Foley et al., 2011; Godfray et al., 2010; Tilman et al., 2002) and potentially burden already stressed planetary boundaries (de Fraiture et al., 2007). A large proportion of the population already lives in water stressed regions, and this proportion is estimated to increase to 50% under the status quo by the end of the century (Wada et al., 2014). The interplay between food security and water resources necessitates the development of new approaches to better understand initiatives that increase agricultural production while reducing our demands on water resources.

Deficit and supplemental irrigation are irrigation practices that limit the application of water use to generally sensitive growth stages to either prevent crop failure, stabilize or increase crop yields, or increase water productivity: the ratio of crop yield to water use (Oweis and Hachum, 2009). Deficit irrigation is generally the limiting of irrigation water for predominately irrigation-fed agriculture, and supplemental irrigation is the practice of introducing limited irrigation for predominately precipitation-fed agriculture (Geerts and Raes, 2009). Deficit and supplemental irrigation, collectively herein referred to as limited irrigation practices, are appropriate initiatives for regions experiencing increased competition for water resources, rising costs associated with water withdrawal and irrigation application, compromised surface water and ground-water availability, and a declining health of associated ecosystems.
(Barrett and Skogerboe, 1980; English, 1990; Fereres and Soriano, 2007; Oweis and Hachum, 2009; Pereira et al., 2002). Evaluating the potential of limited irrigation practices necessitates understanding and evaluating the relationship between seasonal water use and crop yield, which is called the crop-water production function. The crop-water production function provides a framework for evaluating the multi-objective optimization of maximizing agricultural production, water productivity, and profit (Barrett and Skogerboe, 1980; English, 1990; Oweis and Hachum, 2009).

The motivations of the relevant agricultural economy may correspond to and be bounded by limiting factors such as water and land availability, irrigation infrastructure, and the ability of farmers to manage and determine both the quantity and temporal distribution of irrigation water. The crop-water production function allows for the associated agricultural economy to derive optimal solutions acknowledging the bounds of its limiting factors and a weighing of motivations. However, a simple and easily applicable methodology to construct crop- and region-specific crop-water production functions has yet to be developed.

The objectives of this research are twofold: first, to facilitate the development of crop- and region-specific crop-water production functions with limited data that acknowledge the effects of the temporal distribution of water use, and second, to highlight that crop yield is significantly related to the temporal distribution of water use throughout the season. This is necessary for regions interested in evaluating full and limited irrigation practices, and the methodology to determine crop-water production functions with crop coefficients and sensitivity-indices, both discussed in detail later on, is novel. To the best of our knowledge, there is no standard methodology to determine crop- and region-specific crop-water production functions, especially those recognizing the effects of irrigation scheduling. The visualization tool presented herein explicitly presents the entire range of yields associated with each single seasonal water use. The literature on crop-water production functions is largely focused only on developing one-dimensional curves, and mostly without acknowledging the assumptions on the temporal distribution of water use inherent in developing these curves. This is an important concern because crop yields are significantly determined by the temporal distribution of water use and may be more sensitive to the timing of water deficit than the magnitude of the deficit itself (Barrett and Skogerboe, 1980; Howell and Hiler, 1975). Our research and methodology allow decision makers to evaluate their current and proposed irrigation practices by facilitating the development of the associated crop-water production functions and presenting them in the context of the entire space of seasonal water use-crop yield relationships.

The relationship between temporal distributions of water use and crop yield has been quantified generally using sensitivity-index functions (Doorenbos and Kasam, 1979; Geerts and Raes, 2009; Jensen, 1968; Raes et al., 2006; Tafteh et al., 2013). These functions acknowledge the effects of deficits throughout the season during different growth stages. Crop yield is calculated by relating the relative sensitivities of each growth-stage and the relative deficits that befall each growth-stage. Sensitivity-index functions are discussed further in Section 2.3. To the best of our knowledge, these functions have yet to be analyzed over the domain of all possible water distributions, or appropriately integrated with efforts to define the crop-water production function.

To address the above described limitations, we present crop kites, a tool to determine and evaluate the complete space of water use and crop yield relationships where yield is a function of a time-series of water use. A crop kite is the set of points associating to each seasonal water use its range of possible crop yields, and this range relates to the possible temporal distributions of the respective seasonal water use. Our objective is to more appropriately represent the crop-water production function to evaluate economically-optimal irrigation, water and agricultural productivity, and assess the potential of irrigation under water-limited conditions. Subsets of the crop kite are isolated to highlight particular sensitivities and effects on yield given certain assumptions related to the temporal distribution of water use. Crop-water production functions are extracted from the crop kite related to specific irrigation schedules and temporal distributions of water use.

We present the methodology to construct crop- and region-specific crop kites by employing accessible and regionally available data: reference evapotranspiration, crop growth-stage-specific sensitivity-indices, and crop growth-stage-specific crop coefficients to scale reference evapotranspiration to potential evapotranspiration. This is a broadly applicable methodology – where locally-derived crop coefficients and sensitivity-indices are not available, general growth-stage-specific sensitivity-indices and crop coefficients appropriate for regional analyses for 21 different crops were determined by the FAO ( HYPERLINK “ Doorenbos and Kasam, 1979” ), available at http://www.fao.org/nr/water/cropinfo.html. These results have been further evaluated by Moutonnet (2002), and validated by Raes et al. (2006) and Steduto et al. (2012). We significantly improve upon previous efforts to determine crop-water production functions by explicitly integrating the effects of the temporal distribution of water use.

2. Crop-water production functions, crop coefficients, and sensitivity-indices

Previous efforts to investigate the relationship between water use and crop yield have produced both crop-water production functions and sensitivity-index functions. Both functions provide valuable information for the development of irrigation practices under water-limiting conditions. However, in isolation these two interpretations provide only limited perspectives which are inadequate to derive more general conclusions. Crop-water production functions associate with each seasonal water use a single crop yield, and sensitivity index functions showcase an entire range of potential yields associated with each seasonal water use. We employ sensitivity index functions to construct the crop kite and determine crop-water production functions by appropriately collapsing the range of yields for each seasonal water use by assuming different irrigation schedules.

Seasonal water use is defined as the total amount of water used throughout the growing season and crop yield as harvestable yield. Water use may refer to evapotranspiration, transpiration, irrigation water applied, etc. depending on the context, and the methodology developed herein does not necessitate a particular interpretation. However, the case study elaborated on in this paper employs data related to evapotranspiration, and unless otherwise noted, water use will be interpreted as evapotranspiration, and the two may be used interchangeably herein.

2.1. Crop-water production functions

Crop-water production functions relate seasonal water use to crop yield. Previous efforts to represent the crop-water production function have suggested adopting the general shape of, or fitting field data to linear (Barrett and Skogerboe, 1980; English, 1990; Zhang and Oweis, 1999), partially-linear (Doorenbos and Kasam, 1979; English, 1990; Zhang and Oweis, 1999), non-linear polynomial (English, 1990; Hargreaves and Samani, 1984; Kiani and Ab-basi, 2012; Kumar and Khepar, 1980; Oweis and Hachum, 2009; Zhang and Oweis, 1999), rational (Kumar and Khepar, 1980), and logarithmic, as well as generally curvilinear functions (Barrett and Skogerboe, 1980; English, 1990) or combinations thereof (Fereres and Soriano, 2007; Geerts and Raes, 2009; Oweis and Hachum, 2009). For a more complete list of representations as a function
of seasonal water use, we refer the reader to Geerts and Raes (2009). This variety may be due to the different cultivars and climates investigated, as well as the significant disregard of the effects of the temporal distribution of water use. Water deficit may affect crop yield differentially to the sensitivities of the crop growth stages affected (Doorenbos and Kassam, 1979; Jensen, 1968) and may affect crop growth in terms of canopy and vegetative growth, stomatal conductance and photosynthesis, pollination and reproductive growth, root deepening, and canopy senescence (Steduto et al., 2012), and protein and gluten ratios (Tari, 2016).

Non-linear representations generally suggest a curvilinear diminishing return relationship as actual water use approaches potential water use (Barrett and Skogerboe, 1980). The decreasing rate of change relates to two discrete observations, the first relating to field water supply and the second to evapotranspiration and an implicit assumption relating to the temporal distribution of water use. First, the increasing water use or application frequency associated with approaching potential water use may relate to increased inefficiencies such as evaporation, runoff, and deep percolation (Barrett and Skogerboe, 1980; English, 1990). Second, if the temporal distribution of water is assumed to optimize crop yield, then accordingly the most sensitive growth stages would be supplied water preferentially to relay a curvilinear diminishing relationship.

The temporal distribution of water use may significantly affect crop yield and subsequently the same seasonal water use may relate to several significantly different crop yields. Therefore, to appropriately reduce the entire parameter space to a function of seasonal water use, there must be explicit assumptions relating seasonal water use to the temporal distribution of water use, i.e., there must be a function that maps seasonal water uses to temporal distributions of water use. In this way, each seasonal water use maps to a single crop yield. For example, assumptions may be that the distribution of water deficit is optimized for maximum yield, or, the distribution of water deficit is evenly distributed among growth stages. We discuss assumptions relating seasonal water use and temporal distribution of water use in the methodology.

Similarly, to approximate trends from field observations, only a subset of the data observing similar conditions for the temporal distribution of soil moisture should be used. Barrett and Skogerboe (1980) suggest using only the highest yields associated with each seasonal water use. In this way, the crop-water production function represents the relationship of seasonal water use and crop yield assuming optimal temporal distribution of soil moisture. This assumption of optimizing the temporal distribution of water use certainly represents the motivations of the irrigating farmer (Barrett and Skogerboe, 1980), but assumes complete agency over decisions related to the amount and timing of irrigation application. This “irrigation agency” is a function of governance, water accessibility and availability, infrastructure and technology, energy, and costs as they relate to the abstraction, distribution, and transmission of irrigation water. Farmers irrigating with groundwater where costs, energy, resource availability, or regulation are not prohibitive, generally have more facility to tailor both the timing and amount of water than farmers irrigating with public surface water schemes (Dhawan, 1995; Llamas and Custodio, 2003; Shah et al., 2003; Siebert and Döll, 2010; Smilovic et al., 2015). In contrast, assuming maximum irrigation agency may be inappropriate for farmers subject to pre-defined irrigation schedules and amounts. Crop kits provide the context to explicitly consider the range of irrigation agencies available to the farmer to more appropriately evaluate the optimal depth of irrigation water.

2.2. Crop coefficients

Potential water use \( W_p \) is a crop- and climate-specific reference for total crop water use under standard and non-limiting agronomic factors (Doorenbos and Pruitt, 1977). Actual water use \( W_a \) is the total crop water use assuming non-limiting agronomic factors other than the amount and temporal distribution of available soil moisture which may be limiting factors.

Potential water use \( W_p \) is often estimated with the use of a crop coefficient, \( K_c \), a scaling term comparing evapotranspiration to that of a reference surface under similar climatic conditions, often a uniform grass field (Doorenbos and Kassam, 1979). \( K_c \) can be defined as a constant over the entire growing season, or as a non-constant function of time over the crop’s development. Potential water use over the duration of interest is calculated as:

\[
W_p = \int K_c(t) \cdot ET_0(t) \, dt
\]

where \( ET_0 \) is reference evapotranspiration (Doorenbos and Kassam, 1979).

The construction of crop kits requires growth-stage-specific values for potential water use, but does not necessitate the use of crop coefficients and can be substituted for more physically based evapotranspiration estimation methods. Since the objective is to establish a simple and easily applicable methodology to determine crop-water production functions, we elaborate on employing crop coefficients in the case that such other methods or data are not available.

Crop growth can be broadly characterized into growth stages each with potential water use \( W_{pi} \) and actual water use \( W_{ai} \). The temporal distribution of seasonal water use as a time series is represented as a sequence \( W_{a1}, \ldots, W_{ai}, \ldots, W_{an} \) of growth-stage-specific relative water uses. Growth-stage-specific actual water use is the product of growth-stage-specific potential water use and relative water use, namely

\[
W_a = \sum_{i=1}^{n} W_{ai} = \int K_c(t) \cdot ET_0(t) \frac{W_{ai}}{W_{pi}} \, dt
\]

Actual water use is the sum over the entire growing season of all growth-stage-specific actual water uses, namely

\[
W_a = \sum_{i=1}^{n} W_{ai}
\]

\( \frac{W_a}{W_p} \) can be seen as function mapping a sequence of growth-stage-specific relative water uses \( \frac{W_{a1}}{W_{p1}}, \ldots, \frac{W_{an}}{W_{pn}} \) to an overall seasonal relative water use.

2.3. Sensitivity-index functions

Previous efforts (Doorenbos and Kassam, 1979; Jensen, 1968; Kirda, 2002; Raes et al., 2006; Zhang and Oweis, 1999) have acknowledged the effects of the temporal distribution of water deficit by attributing to each growth-stage a sensitivity index \( \lambda_i \) representing the relative sensitivity of the crop to water stress during the growth-stage. Given the non-uniform distribution of growth-stage sensitivities to water deficit, crop growth and associated crop yield will respond to a deficit differently depending on how the deficit is distributed throughout the growing season. It requires two functions, the sensitivity function \( f_i \) that relates a growth-stage-specific deficit to a reduction in crop yield, and the compounding function \( c \) that relates the different growth-stage-specific reductions in crop yield to an overall reduction in crop yield. Previous efforts (Doorenbos and Kassam, 1979; Jensen, 1968; Kirda, 2002; Tafteh et al., 2013) have defined these differently (Table S1) (discussed in the Supplementary material).
Sensitivity-indices $\lambda_i$ have been developed both generally and as a function of cultivar type and climate (Doorenbos and Kassam, 1979; Raes et al., 2006; Zhang and Oweis, 1999). General crop-specific sensitivity-indices are suitable for regional- or large-scale analyses assuming generalized crop conditions and the use of crop cultivars that are well established in the associated growing conditions (Doorenbos and Kassam, 1979). Sensitivity indices provided by Doorenbos and Kassam (1979) have been verified and compared with locally-derived values to show good agreement, and that cultivar-specific derived values will not deviate strongly from the published values (Raes et al., 2006).

Growth-stage-specific relative water use and sensitivity index is calculated in this paper using the product sensitivity function to calculate growth-stage-specific relative yield, namely

$$\left( \frac{Y_i}{Y_R} \right) = 1 - \lambda_i \left( 1 - \frac{W_i}{W_p} \right)$$

where $\left( \frac{Y_i}{Y_R} \right)$ is growth-stage-specific relative yield. The sequence of relative yields is related to an overall relative yield using the multiplicative compounding function, namely

$$\frac{Y_o}{Y_P} = \prod_{i=1}^{N} \left( \frac{Y_i}{Y_R} \right) = \prod_{i=1}^{N} \left( 1 - \lambda_i \left( 1 - \frac{W_i}{W_p} \right) \right)$$

where potential yield $Y_P$ is the yield associated with potential water use $W_p$, and $Y_o$ is actual yield. In this way, $\frac{Y_o}{Y_P}$ can be seen as function mapping a sequence of growth-stage-specific relative water uses $\left( \frac{W_1}{W_p}, \ldots, \frac{W_N}{W_p} \right)$ to an overall seasonal relative yield. The multiplicative compounding function was recommended by Raes et al. (2006) as it was shown to produce more reliable results than other compounding functions compared in their analysis (discussed in the Supplementary material).

3. Methodology

3.1. General crop kite

Crop kites are constructed by mapping the space of all possible distributions of seasonal water use to the plane relating relative seasonal water use and relative yield. We first construct the space $D$ representing all possible distributions of seasonal water use. Each growth-stage is associated with a minimum relative water use $T_i$ below which the crop experiences terminal moisture stress, defined as the minimum threshold for soil moisture below which crop growth terminates and the overall yield is zero. The space $D$ is then the allowable distribution of deficits that result in non-zero overall yield. For example, given a crop with two growth stages such that growth-stage 1 has terminal moisture stress $T_1=0.5$ and growth-stage 2 has terminal moisture stress $T_2=0.4$, the space $D$ is $[0.5, 1] \times [0.4, 1]$. A visual representation of $D$ is provided in Fig. 1. However, to map elements from this domain, it is necessary to discretize $D$, namely

$$D = \left\{ \left( \frac{W_0}{W_p} \right)_i \left| W_o \in \left\{ T_i + x \Delta d \right\}, x \in \{0, 1, \ldots, 1 - \frac{T_i}{\Delta d} \} \right. \right\}$$

where $\Delta d \geq 0$ is the discretization step.

Every member of $D$ is associated with a relative seasonal water use and relative overall yield by interpreting $\frac{W_0}{W_p}$ and $\frac{Y_o}{Y_P}$ as functions of $D$. The crop kite is the set of associated relative seasonal water uses and relative overall yields for all possible distributions of seasonal water use, namely

$$\text{Crop kite} = \left\{ \left( \frac{W_0}{W_p}(d), \frac{Y_o}{Y_P}(d) \right) \left| d \in D \right. \right\}$$

The sequence of steps, or work flow, to map an element from $D$, the space of all possible distributions of seasonal water use, to an element on the crop kite is illustrated in Fig. 1. Each temporal distribution of water use in $D$ is associated with both (1) a total relative seasonal water use using the $K_c$ crop coefficients and reference evapotranspiration $ET_0$, and (2) a relative crop yield using the sensitivity-indices $\lambda_i$, sensitivity functions $f_i$, and compounding function $c$.  

3.2. Example: winter wheat crop kite

To demonstrate the efficacy of the crop kite method in determining crop-water production functions, we construct the crop kite for winter wheat using crop-specific sensitivity indices provided by the FAO (Doorenbos and Kassam, 1979) and validated by Raes et al. (2006). Wheat covers more land surface globally than
any other cultivated crop (Portmann et al., 2010) and is the third largest produced in terms of weight (Steduto et al., 2012). Winter wheat has been studied extensively for use with limited irrigation practices (Ilbeyi et al., 2006; Iqbal et al., 2014; Jin et al., 2014; Mahmoud et al., 2015; Oweis et al., 1998; Rezavandiejad et al., 2014; Salemi et al., 2011; Tafteh et al., 2013; Tavakkoli and Oweis, 2004; Zhang et al., 2013), and compared with spring varieties it may produce higher yields and be more resilient in moisture limiting conditions (FAO, 2016; Steduto et al., 2012). In general, the growing period for winter wheat may range from 180 to 315 days, and evapotranspiration from 450 to 650 mm depending on climate and cultivar (Steduto et al., 2012). Raes et al. (2006) previously evaluated the sensitivity indices of Doorenbos and Kassam (1979), as well as provided an analysis on the performance of various compound functions as they relate to winter wheat specifically verified with field observations from northern Tunisia with a Mediterranean climate typical of the West Asia–North Africa region (Raes et al., 2006). The data used to construct the example crop kite, including crop coefficients, sensitivity indices, and reference evapotranspiration are derived from Raes et al. (2006) and presented in Tables 4 and 5, and Fig. 2. Notice that the stages referring to crop coefficients and sensitivity indices need not be the same. Unless otherwise noted, for the following example “growth stages” refer to those associated with the sensitivity indices. Crop growth stages are means of partitioning an uninterrupted process of crop development for the purpose of recording growth, or to categorize segments of similar water use or sensitivities. Water use may remain relatively constant once there is significant leaf mass and canopy cover, while the growing process may shift from stem elongation to flowering to grain filling, each with a different response and sensitivity to water deficit.

Explicitly, \( K_c \) as a function of time \( t \) is defined as:

\[
K_c(t) = \begin{cases} 
0.4 & t \in [0, 30], \\
0.4 \left(1 - \frac{t - 30}{80}\right) + 1.15 \left(\frac{t - 30}{80}\right) & t \in [30, 110], \\
1.15 & t \in [110, 150], \\
1.15 \left(1 - \frac{t - 150}{30}\right) + 0.25 \left(\frac{t - 150}{30}\right) & t \in [150, 180]. 
\end{cases}
\]

Daily reference evapotranspiration representative of the 1999–2000 growing season (December 29–June 25) is derived from the 10-days reference evapotranspiration from Raes et al. (2006) by assuming equal daily evapotranspiration during each 10-days period

\[
ET_0(t) = 0.000105t^2 + 0.0113t + 1.42
\]

Fig. 2 presents the crop coefficient function, reference evapotranspiration function and quadratic fit, and the function representing daily potential water use for winter wheat.

Fig. 3 illustrates an example of actual water use throughout the growing season by filling in for each of the five growing periods, the ratio of actual water use to potential water use; accordingly, the associated deficits are represented in the unfilled upper space. We set the terminal moisture stress for each growth-stage to be 0.40, generalizing the terminal moisture stress for each growth-stage indicated for winter wheat by the FAO (2016). In practice, each growth-stage may be attributed with a unique terminal moisture stress, and we set them equal so as to not unnecessarily complicate the example. We set the discretization step to \( \Delta d=0.02 \).
Our domain \( D \) is then

\[
D = \left\{ \left( \frac{W_{a1}}{W_p}, \frac{W_{a2}}{W_p}, \frac{W_{a3}}{W_p}, \frac{W_{a4}}{W_p}, \frac{W_{a5}}{W_p} \right) \mid \frac{W_{a5}}{W_p} \in \{0.40, 0.42, \ldots, 1\} \right\}
\]

We map every element in \( D \) to its seasonal relative water use by taking the ratio of actual water use and potential water use, where \( K_c \) and \( E_{T0} \) are as defined explicitly above. Note that the different lengths of growth stages associated with the crop coefficients and sensitivity-indices must be dealt with carefully in the following calculation.

Actual water use:

\[
W_o = \int_1^{10} 0.4 \cdot E_{T0} \cdot \frac{W_{a1}}{W_p} \, dt + \int_{11}^{30} 0.4 \cdot E_{T0} \cdot \frac{W_{a2}}{W_p} \, dt + \int_{31}^{105} 0.4 \left( 1 - \frac{t - 30}{80} \right) + 1.15 \left( \frac{t - 30}{80} \right) \cdot ET_o \cdot \frac{W_{a3}}{W_p} \, dt
\]

\[
+ \int_{106}^{130} 0.4 \left( 1 - \frac{t - 30}{80} \right) + 1.15 \left( \frac{t - 30}{80} \right) \cdot ET_o \cdot \frac{W_{a4}}{W_p} \, dt
\]

\[
+ \int_{131}^{150} 1.15 \cdot ET_o \cdot \frac{W_{a4}}{W_p} \, dt + \int_{151}^{180} 1.15 \cdot ET_o \cdot \frac{W_{a5}}{W_p} \, dt
\]

Potential water use:

\[
W_p = \int_1^{180} K_c \cdot ET_o \, dt
\]

Similarly, we map every element in \( D \) to its seasonal relative yield:

\[
Y_o = \left( 1 - 1.0 \left( 1 - \frac{W_{a1}}{W_p} \right) \right) \left( 1 - 0.2 \left( 1 - \frac{W_{a2}}{W_p} \right) \right) \left( 1 - 0.6 \left( 1 - \frac{W_{a3}}{W_p} \right) \right) \left( 1 - 0.5 \left( 1 - \frac{W_{a4}}{W_p} \right) \right) \left( 1 - 0.2 \left( 1 - \frac{W_{a5}}{W_p} \right) \right)
\]

The crop kite is the space of all associated relative water uses and relative yields, where association implies that an element of \( D \) maps to both the relative water use and relative yield.

3.3. Developing crop kite subsets and crop-water production functions with weak and strong assumptions

The crop kite can be collapsed into smaller subsets and eventually into crop-water production functions given certain assumptions about the distribution of seasonal water use. Subsets and the crop-water production function demonstrate how crop yield potential may be affected assuming scenarios related to the distribution of water use. Without explicit assumptions relating to the temporal distribution of water use, previous efforts have merely patched together disparate subsections of the resulting solution space, trivializing the realities of the relationship between water use and yield. With crop kites, two classes of assumptions can be used to develop subsets and extract crop-water production functions, namely “weak” and “strong” assumptions accordingly. Weak assumptions characterize only sections of the water distribution for the growing season; for example, there is no water deficit in the first growth-stage. Strong assumptions characterize the entire growing season; for example, the water deficit is evenly distributed between the growth stages. Explicitly, weak assumptions restrict the domain \( D \), and strong assumptions define a function that maps seasonal water uses to temporal distributions of water use. Developing crop-water production functions explicitly acknowledging constraints associated with the proposed irrigation schedule allows for a more appropriate evaluation of the potential of a limited irrigation practice given the realities of the agricultural economy.

In the following section, we present subsets of the crop kite for winter wheat associated with the following weak (W) assumptions and crop-water production functions associated with the following strong (S) assumptions:

- W1.1 There is maximum deficit in growth-stage \( i \).
- W2.1 There is no deficit in growth-stage \( i \).
- S1. The distribution of deficit is optimized for maximum yield.
- S2. The deficit is evenly distributed.
- S3. Water use is allocated preferentially from the first to fifth growth-stage.

4. Results and discussion

4.1. Crop kite

The crop kite acknowledges the timing of water deficit on crop yield: every point on the crop kite is related to at least one temporal distribution of a seasonal water use. The crop kite for winter wheat presented in Fig. 4 illustrates the potential range of relative crop yield related to each relative seasonal water use; for example, 80% seasonal water use relates to between ~20% and 90% of relative crop yield. Fig. 4 presents a variety of examples illustrating the effects of the temporal distribution of water use on crop yield.

4.2. Subsets

4.2.1. Growth-stage-specific assumptions

It may be of interest to make assumptions related to specific growth stages, and evaluate the response of the crop kite. To further explain, we elaborate on assumptions W1.1 and W2.1, illustrated in the top row of Fig. 5 in the context of the encompassing crop kite, where maximum water deficit (water deficit just above terminal water stress) and no water deficit respectively occur in growth-stage 1. Maximum water deficit in growth-stage 1 relates to 0.6% of potential water use, and a 60% reduction in potential yield. Securing water-stress free conditions in the initial growth-stage secures crop yield to a minimum relative yield of 35% assuming no terminal moisture stress in the subsequent growth stages, that is, increasing the minimum yield by 21% as compared to the encompassing crop kite. These assumptions highlight the significant benefits from the controlled application of water in the initial growth stages under soil moisture-limited conditions. Assumptions W1.1–W1.5 and Figs. 5 and 6a illustrate the scenario where maximum water deficit occurs in a particular growth-stage, for example, either from insufficient precipitation or unavailable, insufficiently allocated, or poorly scheduled irrigation water. Assumptions W2.1–W2.5 and Fig. 5 and 6b illustrate the scenario where water use meets potential water use for a particular growth-stage. Evaluating assumptions relating to one or multiple growth stages can facilitate preliminary decision-making related to irrigation scheduling, and provide a partial collapse of the crop kite to understand how combinations of stresses across different growth stages may limit or support potential options for crop-water production functions, discussed in the following section.

4.2.2. Crop-water production functions

Collapsing the crop kite into crop-water production functions is valuable in demonstrating how yield may respond to changes
in water use assuming particular distributions of water use. This method of constructing the crop-water production function improves upon previous efforts by associating the temporal distribution of seasonal water use to each point along the function. Examples of different functions associated with the aforementioned strong assumptions are discussed below and presented in Fig. 7, which are available online as interactive graphs connecting each point along the curve with its temporal distribution (links in the figure caption).

Assumption S1 considers the scenario where water use is distributed to maximize crop yield (Fig. 7a). This extracted crop-water production function agrees with previous efforts defining the shape as a diminishing curvilinear function. This assumes maximum irrigation agency and subsequently the ability to intimately manage irrigation water, both in terms of amount and temporal distribution. This scenario may best represent farmers irrigating with sufficiently available groundwater abstracted with privately owned wells.

Assumption S2 considers the scenario where water deficit is distributed evenly between the growth stages (Fig. 7b). This scenario may represent agricultural communities employing a shared irrigation system with limited water and a fixed rotating schedule.

Assumption S3 considers the scenario where water use is distributed preferentially from the first to the fifth growth stages (Fig. 7c). This scenario may represent an agricultural community with a shared irrigation system with limited water and without appropriation limits. The scenario may encourage opportunistic behavior and is vulnerable to encouraging a pumping or abstraction race towards the beginning of the season (the example illustrated assumes that enough water is available to avoid terminal moisture stress in later stages). This may result in all farmers meeting potential water use until the water has been fully appropriated. However, as compared to adopting a fixed rotation schedule associated with assumption S2 and distributing the deficit evenly, assumption S3 always simulates higher yields. Depending on the costs associated with adopting certain irrigation distribution patterns and the related benefits, such as the costs and benefits of adopting and enforcing regulations related to irrigation distribution, the agricultural economy can compare the collapsed crop

![Fig. 4. Crop kite for winter wheat. Relative water use is on the x-axis and relative yield is on the y-axis. Each point on the crop kite is associated with a temporal distribution of water use, visualized similarly to Fig. 3. The six highlighted points were chosen to provide a diversity of examples, described as follows: (a) 30% deficit evenly distributed; (b) 22% deficit distributed as 0%, 10%, 20%, 30%, and 40%; (c) 0% deficit; (d) 10% deficit evenly distributed; (e) 23% deficit distributed as 30%, 10%, 0%, 50%, and 20%; and (f) 60% deficit evenly distributed.](image-url)
Fig. 5. (W1.i) Subsets of the crop kite that assumes a maximum deficit in a growth-stage i. (W2.i) Subsets of the crop kite that assumes no water deficit in growth-stage i. The lighter background for each subset is the full crop kite as shown in Fig. 4.

kites to best evaluate the most appropriate and cost-optimized developments.

There have been significant efforts to characterize the shape of the crop-water production function in the literature, as discussed in Sections 1 and 2.1, and assumption S1 agrees with characterizing the shape as a diminishing curvilinear function assuming maximum irrigation agency. However, none of the presumed variety includes the resulting shapes of assumptions S2 and S3, which do not conform to the general linear or diminishing curvilinear fits. It can thus be generalized that previous efforts largely attempted
Fig. 6. (a) Subsets of the crop kite that assume a maximum deficit in the associated growth-stage; this is an overlaying of the figures in the left column of Fig. 5. (b) Subsets of the crop kite that assume no deficit in the associated growth-stage; this is an overlaying of the figures in the right column of Fig. 5.
Crop–water production functions determined with assumptions related to the irrigation schedule are most appropriate for evaluating contexts where irrigation is a significant component of crop water use, such as in arid and semi-arid environments. Determining the crop–water production function for contexts evaluating supplemental irrigation for mainly precipitation-fed agriculture necessitates acknowledging the inherent variability in the distribution of precipitation. However, this requires integrating the discussed methodology with a crop growth–soil water balance model to articulate the response of soil moisture to precipitation, irrigation, and crop evapotranspiration. Nevertheless, consistent with our motivations for evaluating limited irrigation practices with limited data, crop kites can still be an accessible tool for preliminary evaluations.

Irrigation decisions may also relate to and be supported by other initiatives to increase agricultural and water productivity including nutrient- and pest-management; the automation and mechanization of monitoring and decision making, and the adoption of precision technologies; crop selection and breeding through both conventional and molecular techniques; fostering soil health and limiting erosion through different tillage practices and crop rotations; and shifting planting dates. Further increases in productivity and quality will necessitate exploring a diverse portfolio of technologies and initiatives, and adopting those most appropriate for the associated agroclimatic, socioeconomic, and environmental conditions.

Isolating subsets with weak assumptions related to the average or problematic distribution of evapotranspiration illustrates the potential effects of not addressing deficits not satisfied by precipitation during different or collective growth stages. Crop–water production functions relating to potential irrigation schedules appropriate for the agricultural economy may relate to worst-case scenarios, appropriate for evaluating irrigation buffering during times of climatic variability.

5. Conclusions

Deficit and supplemental irrigation may be appropriate initiatives to increase agricultural and water productivity, and to buffer the vulnerabilities of precipitation-fed agriculture. Crop kites are a tool to evaluate the potential of irrigation practices under water-limiting conditions, wherein crop yield is determined both by the amount and temporal distribution of water use. Previous efforts have attempted to characterize the crop–water production function as function of seasonal water use while not acknowledging the effects of the temporal distribution of water use, or simply assuming maximum irrigation agency. In this way, the interpretation compromises the very capacity of the function to evaluate the multi-objective optimization that motivated its development. Crop kites provide the context to evaluate the optimal depth of irrigation water by acknowledging the variation in crop yields associated with each seasonal water use.

The crop- and region-specific crop kite is the complete space of the seasonal water use-crop yield relationship, and a diagnostic tool if coupled with an understanding of the current or proposed temporal distributions of water use. Previous studies have made efforts to determine generalized fits for crop-water production functions, however, each shape is inherently associated with an assumption on the temporal distribution of seasonal water use. The assumed fit of crop-water production functions to diminishing curvilinear functions, inherently assuming an optimized distribution, may be aligned with the motivations of maximizing possible agricultural and water productivity, but is only appropriate if the region and irrigation practitioners have the capacity to manage exactly the timing and amount of irrigation water. The graphical representation of the crop kite is a corollary of constructing the crop

![Fig. 7. Crop-water production functions (S1) maximum yield: deficit optimized for maximum yield, (S2) deficit evenly distributed, and (S3) water use allocated preferentially from first to fifth growth-stage. Interactive plots associating with each point its temporal distribution of water use are available at (a) https://plot.ly/~Mikhail/3052, (b) https://plot.ly/~Mikhail/3068, and (c) https://plot.ly/~Mikhail/3071.](image-url)
kite to then tease apart the appropriate and representative one-dimensional crop-water production functions.

In this study, we provided the following novel contributions:

1. Crop kites are a tool to easily determine appropriate crop-and region-specific crop-water production functions that depend on the temporal distribution of water use and are constructed using generally accessible data: reference evapotranspiration, and growth-stage-specific sensitivity-indices and crop coefficients.

2. Crop-water production functions are not necessarily diminishing curvilinear functions (Fig. 7b and c), and the shape depends on the associated assumptions relating to irrigation agency and temporal distribution of water use, made explicit in this paper.

3. Crop kites illustrate the non-unique relationship between seasonal water use and crop yield and acknowledge both irrigation agency and the temporal distribution of water use.

4. Subsets of the crop kite illustrate how assumptions relating to particular growth stages affect the seasonal water use-crop yield relationship.

The evaluation of agricultural production under water-limiting conditions necessitates acknowledging the range of crop yields associated with a single seasonal water use. Agricultural communities are able to determine their current position on the crop kite, and evaluate the necessary increases in either seasonal water use or irrigation agency to meet certain production goals. Assuming temporal distributions that optimize crop yield, in isolation of the encompassing crop kite, represents only an agricultural economy where farmers can immediately manage both the amount and timing of water use. This assumption in the context of the crop kite, however, allows the relevant agricultural community to evaluate the necessary expansions in irrigation agency and associated costs. Crop kites are a tool to more appropriately evaluate irrigation scheduling, water management, economically optimal irrigation depths, water and agricultural productivity, and assessing deficit and supplemental irrigation.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.adwatres.2016.09.010.

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